

5-13 Using Factoring to Solve Problems

Objective: To solve problems by writing and factoring quadratic equations.

Example 1 Find two consecutive positive odd integers whose product is 143.

Solution

Step 1 The problem asks for two consecutive positive odd integers.

Step 2 Let n = the first integer. Then $n + 2$ = the second integer.

Step 3 Use the facts in the problem to write an equation. $n(n + 2) = 143$

Step 4 Solve the equation. $n^2 + 2n - 143 = 0$
 $(n + 13)(n - 11) = 0$

$$n + 13 = 0 \quad \text{or} \quad n - 11 = 0$$

$$n = -13 \quad \quad \quad n = 11$$

You are to find positive odd integers, so reject -13 . If $n = 11$, then $n + 2 = 13$.

Step 5 *Check:* $11 \times 13 = 143$. The integers are 11 and 13.

Example 2 Originally a rectangle was 8 cm by 17 cm. When both dimensions were decreased by the same amount, the area of the rectangle decreased by 66 cm^2 . Find the dimensions of the new rectangle.

Solution

Step 1 The problem asks for the dimensions of the new rectangle.

Step 2 Let x = the amount by which each dimension is decreased. Make a sketch. The new dimensions are $17 - x$ and $8 - x$.

Step 3
$$\begin{array}{l} \text{(Original)} \\ \text{area} \end{array} - \begin{array}{l} \text{(Decrease)} \\ \text{in area} \end{array} = \begin{array}{l} \text{(New)} \\ \text{area} \end{array}$$

$$(17 \cdot 8) - 66 = (17 - x)(8 - x)$$

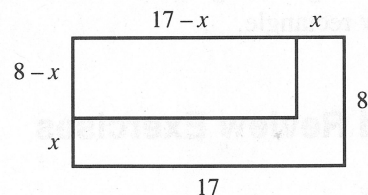
Step 4
$$\begin{aligned} 136 - 66 &= 136 - 25x + x^2 \\ 70 &= 136 - 25x + x^2 \\ 0 &= 66 - 25x + x^2 \\ 0 &= x^2 - 25x + 66 \\ 0 &= (x - 3)(x - 22) \end{aligned}$$

$$x - 3 = 0 \quad \text{or} \quad x - 22 = 0$$

$$x = 3 \quad \quad \quad x = 22$$

Step 5 Check in the words of the problem and you'll see that you must reject 22.

The new rectangle is 14 cm long and 5 cm wide.



CAUTION

A solution of an equation may not satisfy some of the conditions of the problem. You reject solutions of an equation that do not make sense for the problem.

5-13 Using Factoring to Solve Problems (continued)

Solve.

1. If a number is added to its square, the result is 72. Find the number.
2. If a number is subtracted from its square, the result is 90. Find the number.
3. A positive number is 56 less than its square. Find the number.
4. A negative number is 56 less than its square. Find the number.
5. Find two consecutive negative integers whose product is 72.
6. Find two consecutive positive even integers whose product is 120.
7. The sum of the squares of two consecutive positive odd integers is 202. Find the integers.
8. The sum of the squares of two consecutive negative odd integers is 130. Find the integers.
9. The length of a rectangle is 5 cm greater than its width. Find the dimensions of the rectangle if its area is 126 cm^2 .
10. The length of a rectangle is 8 cm less than twice its width. Find the dimensions of the rectangle if the area is 120 cm^2 .
11. Find the dimensions of a rectangle whose perimeter is 40 m and whose area is 96 m^2 . (*Hint*: Let the width be w . Use the perimeter to find the length in terms of w .)
12. Find the dimensions of a rectangle whose perimeter is 52 m and whose area is 160 m^2 .
13. Originally the dimensions of a rectangle were 12 cm by 7 cm. When both dimensions were decreased by the same amount, the area of the rectangle decreased by 34 cm^2 . Find the dimensions of the new rectangle.
14. Originally a rectangle was twice as long as it was wide. When 5 m was subtracted from its length and 3 m subtracted from its width, the resulting rectangle had an area of 55 m^2 . Find the dimensions of the new rectangle.

Mixed Review Exercises

Simplify.

- | | | |
|---------------------------------------|--|--|
| 1. $(6ab^2)(3a^2b)$ | 2. $(4a^2)^3$ | 3. $2a(3 - 2a)$ |
| 4. $(8r)\left(\frac{1}{4}rs^2\right)$ | 5. $(2by^2)^2$ | 6. $\left(\frac{1}{6}\right)(18n - 30p)$ |
| 7. $(2a + 3)(2a^2 + 3 - 5a)$ | 8. $(-20x - 15y)\left(-\frac{1}{5}\right)$ | 9. $(5m - 3)^2$ |

Factor completely.

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|------------------------|---------------------------|-----------------------|
| 10. $6m^2 + 2m - 20$ | 11. $28a^3 - 7ab^2$ | 12. $15n^2 + 14n - 8$ |
| 13. $y^4 - y^3 - 6y^2$ | 14. $12m^2 + 13mn + 3n^2$ | 15. $8x^2 - 14x + 3$ |