

4-2 Adding and Subtracting Polynomials

Objective: To add and subtract polynomials.

Vocabulary

Monomial An expression that is either a numeral, a variable, or the product of a numeral and one or more variables. For example: 13, m , $8c$, $2xy$, $5p^2q$.

Coefficient In the monomial $9x^2y^3$, 9 is the coefficient, or numerical coefficient.

Similar, or like, terms Two monomials that are exactly alike or the same except for their numerical coefficient. For example, $-3xy$ and $7xy$ are similar.

Constant monomial or constant A numeral such as 7.

Polynomial A sum of monomials. For example, $x^2 + 3x + y^2 + 2$.

Binomial A polynomial of only two terms. For example, $2x + 5$.

Trinomial A polynomial of only three terms. For example, $a^2 + 2ab + b^2$.

Simplified form, or simplest form, of a polynomial A polynomial which has no two of its terms similar.

CAUTION When a monomial does not have a written numerical coefficient, remember that its coefficient is 1. For example, $x^6y^2 = 1x^6y^2$.

Example 1 Simplify $-5x^3 + 2x^2 + x^2 + 7x^3 - 4$.

Solution

$$\begin{aligned} -5x^3 + 2x^2 + x^2 + 7x^3 - 4 &= (-5x^3 + 7x^3) + (2x^2 + x^2) - 4 \\ &= (-5 + 7)x^3 + (2 + 1)x^2 - 4 \\ &= 2x^3 + 3x^2 - 4 \end{aligned}$$

Simplify.

1. $2x - y + 3x - 2y$

2. $7m - 5n - 2m + n$

3. $4x^2 - 3x - 2x^2 + 7x - 2$

4. $n^2 - 3n - 5n^2 + 7n + 6n^2$

5. $a^2 + 2ab - 5ab + 4a^2$

6. $x^2y - y^3 - 8x^2y + 5y^3$

7. $a^2b - 2ab^2 + 5a^3 - 3a^2b$

8. $-6xy^2 + 5x^2y - x^3 + xy^2 + 3x^3 - 2x^2y$

Example 2 Add $2x^2 + 5xy - 6y^2$ and $8x^2 + 6xy + y^2$.

Solution 1 First group similar terms and then combine them.

$$\begin{aligned} (2x^2 + 5xy - 6y^2) + (8x^2 + 6xy + y^2) &= (2x^2 + 8x^2) + (5xy + 6xy) + (-6y^2 + y^2) \\ &= 10x^2 + 11xy - 5y^2 \end{aligned}$$

Solution 2

$$\begin{array}{r} 2x^2 + 5xy - 6y^2 \\ 8x^2 + 6xy + y^2 \\ \hline 10x^2 + 11xy - 5y^2 \end{array} \quad \left\{ \begin{array}{l} \text{You can also align similar} \\ \text{terms vertically and add.} \end{array} \right.$$

4-2 Adding and Subtracting Polynomials (continued)**Vocabulary**

Degree of a variable in a monomial The number of times that the variable occurs as a factor in a monomial. For example, in $7x^3y$, the degree of x is 3, and the degree of y is 1.

Degree of a monomial The sum of the degrees of its variables. For example, the degree of $8x^2y^3$ is 5. The degree of any nonzero constant monomial, such as 10, is 0.

Degree of a polynomial The greatest of the degrees of its terms after it has been simplified. For example, the degree of $-5x^3 + 2x^2 + x^2 + 5x^3 - 4$ is 2, since it can be simplified to $3x^2 - 4$.

Add.

$$\begin{array}{r} 9. \quad 3a - 1 \\ \quad 4a + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 4n + 2 \\ \quad -2n - 5 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 2x - 3y \\ \quad -2x + 6y \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 5n - 2p \\ \quad -3n + 5p \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 4x - 5y + 3 \\ \quad -2x + 7y + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 2a - 3b - 6 \\ \quad 3a - b + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 6x^2 - 3x + 2 \\ \quad 2x^2 + x - 5 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 5 - 2n - 6n^2 \\ \quad -3 + n - 2n^2 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 4c^2 - 3cd - 5d^2 \\ \quad -c^2 + 6cd - 2d^2 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 6a^2 - 2ab \\ \quad -2a^2 + 5ab - b^2 \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad 3x - 2y - 5z + 1 \\ \quad 2x + y - 3z \\ \quad 3y + z + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 6a - 2b + 4 \\ \quad 3a - 5c - 1 \\ \quad -a - b + 6c + 5 \\ \hline \end{array}$$

Example 3 Subtract $-x^2 + 5xy + 6y^2 - 3$ from $3x^2 - 6xy - 2y^2 - 5$.

Solution 1 Add the opposite of $(-x^2 + 5xy + 6y^2 - 3)$ to $3x^2 - 6xy - 2y^2 - 5$.
 $(3x^2 - 6xy - 2y^2 - 5) - (-x^2 + 5xy + 6y^2 - 3) =$
 $(3x^2 - 6xy - 2y^2 - 5) + (x^2 - 5xy - 6y^2 + 3) = 4x^2 - 11xy - 8y^2 - 2$

Solution 2 You can also align similar terms vertically.

$$\begin{array}{r} 3x^2 - 6xy - 2y^2 - 5 \\ -(-x^2 + 5xy + 6y^2 - 3) \\ \hline \end{array} \longrightarrow \begin{array}{l} \text{Change to the} \\ \text{opposite signs} \\ \text{and add.} \end{array} \longrightarrow \begin{array}{r} 3x^2 - 6xy - 2y^2 - 5 \\ x^2 - 5xy - 6y^2 + 3 \\ \hline 4x^2 - 11xy - 8y^2 - 2 \end{array}$$

21-30. In Exercises 9-18, subtract the lower polynomial from the upper one.

Mixed Review Exercises

Simplify.

1. -2^3

2. $(-3)^2$

3. $2^2 + 3^2$

4. $(2 + 3)^2$

Solve.

5. $3(y + 2) - 2 = 2(4 - y)$

6. $10 = 2(n + 3)$

7. $4(x - 10) = 13 - 3(2x + 1)$

8. $-\frac{2}{5}(n + 3) = 10$

9. $c - 2 = |1 - 8|$

10. $\frac{3}{4}(2y - 6) = y - 7$